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BRIEF COMMUNICATIONS

TORQUE EXERTED ON A SLOWLY ROTATING ECCENTRICALLY POSITIONED SPHERE WITHIN AN INFINITELY LONG CIRCULAR CYLINDER

T. Greenstein and G. L. Schiavina†

Department of Chemical Engineering and Chemistry, New Jersey Institute of Technology, Newark, NJ 07102, U.S.A.

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1. INTRODUCTION

The torque required to maintain the steady rotation of a sphere in an unbounded, incompressible, viscous liquid at small angular Reynolds numbers is well known (Lamb 1945). The above study is severely limited by the assumption of an unbounded medium, for no such thing really exists. In practical applications the fluid does not extend to infinity but, rather, is bounded externally, usually by a circular cylinder. Knowledge of the increased torque arising from the presence of the cylinder walls is important, for example, in the theory of rotational viscometers and in determining the power requirements for the agitation of highly viscous Newtonian fluids in the laminar regime.

Haberman (1961) and, independently, Brenner & Sonshine (1964) studied the problem of a sphere (radius $= a$) rotating slowly with constant angular velocity Ω_3 about an axis lying along the longitudinal Z-axis of an infinitely long circular cylinder of radius R_0, the viscous fluid being at rest at $|Z| = \infty$. By utilizing the expressions derived by Greenstein (1967), we shall extend this problem to the general case where the sphere may be placed *eccentrically* within the cylinder and may rotate with an arbitrary constant angular velocity, $\boldsymbol{\Omega}$, *about any axis.*

2. DESCRIPTION OF THE PROBLEM

Consider the slow rotation of a spherical particle rotating with an arbitrary constant angular velocity through a viscous incompressible fluid confined within an infinitely long circular cylinder. The sphere of radius a rotates with an arbitrary constant angular velocity, $\boldsymbol{\Omega} = \mathbf{i}\Omega_1 + \mathbf{j}\Omega_2 + \mathbf{k}\Omega_3$, relative to the cylinder wall. The cylinder radius is R_0 and the center of the sphere is situated at a distance b from the cylinder axis in the $\mathbf{i}$-direction, as shown in figure 1.

It is assumed that the fluid motion is governed by the creeping motion and continuity equations,

$$\mu \nabla^2 \mathbf{V} = \nabla p \tag{1}$$

and

$$\nabla \cdot \mathbf{V} = 0. \tag{2}$$

$\mathbf{V}$ is the fluid velocity with respect to a coordinate system having its origin at the sphere center, p is the dynamic pressure and μ the fluid viscosity. The boundary conditions which define the fluid

†Present address: Ecodyne Corporation—Unitech Division, Union, NJ 07083, U.S.A.

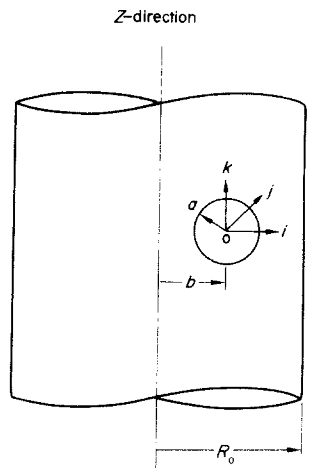

Figure 1. Sphere rotating in a circular cylinder containing a viscous fluid.

velocity field **V** are (i) at fluid–solid interfaces there is no relative motion; hence

$$\mathbf{V} = \mathbf{\Omega} \times \mathbf{r} \quad \text{at } r = a \tag{3}$$

and

$$\mathbf{V} = \mathbf{0} \qquad \text{at } R = R_0, \tag{4}$$

where **r** is measured from the center of the sphere; (ii) at large distances from the disturbing influence of the sphere, $|z| = \infty$, the fluid is at rest, i.e.

$$\mathbf{V} \rightarrow \mathbf{0} \text{ as } z \rightarrow \pm\infty. \tag{5}$$

Use of the creeping motion equations restricts the validity of the final results to situations in which the angular particle Reynolds number, $4a^2|\mathbf{\Omega}|/\nu$, is small; ν is the kinematic viscosity.

The above boundary-value problem can be solved by a technique of successive approximations known as the method of reflections (see Happel & Brenner 1965). The results presented below are only valid for $a \ll R_0 - b \leqslant R_0$.

3. RESULTS AND DISCUSSION

The following expression was obtained for the torque required to maintain the slow, steady, arbitrary rotation of an eccentrically situated sphere in a viscous liquid bounded externally by an infinitely long circular cylinder.

$$\begin{aligned}\mathbf{T} = &-\mathbf{i}8\pi\mu a^3\Omega_1\left[1 - A(\beta)\left(\frac{a}{R_0}\right)^3 - O\left(\frac{a}{R_0}\right)^5\right]\\ &-\mathbf{j}8\pi\mu a^3\Omega_2\left[1 - B(\beta)\left(\frac{a}{R_0}\right)^3 - O\left(\frac{a}{R_0}\right)^5\right]\\ &-\mathbf{k}8\pi\mu a^3\Omega_3\left[1 - C(\beta)\left(\frac{a}{R_0}\right)^3 - O\left(\frac{a}{R_0}\right)^5\right].\end{aligned} \tag{6}$$

The functions $A(\beta)$, $B(\beta)$, and $C(\beta)$ derived by Greenstein (1967) have been evaluated numerically by Schiavina (1973), for various values of the parameter ($\beta = b/R_0$) and the results obtained are tabulated in table 1. Refer to Schiavina (1973) for additional details.

Various particular cases may be derived from our general work by making the following substitutions in [6]. When the sphere rotates only about the **i**-direction, set $\Omega_2 = \Omega_3 = 0$. When the sphere rotates only about the **j**-direction, set $\Omega_1 = \Omega_3 = 0$; and when the sphere rotates only about the **k**-direction, set $\Omega_1 = \Omega_2 = 0$. When the sphere is situated at the center of the cylinder, $\beta = 0$, and the appropriate corresponding value of $A(\beta)$, $B(\beta)$, and/or $C(\beta)$ is read from table 1. As would be expected from symmetry considerations, it has been found that the value of $A(\beta) = B(\beta)$ at $\beta = 0$.

Verification of a portion of the results presented in the preceding section is provided by the theoretical value obtained by Haberman (1961) and, independently, by Brenner & Sonshine (1964) for the particular case of the rotation of a sphere about the longitudinal axis of an infinitely long circular cylinder. From table 1, for the special case of a sphere whose center is situated on the cylinder axis ($\beta = 0$), $C(\beta) = -0.796811$. This is in very good agreement with the value of -0.79680 obtained by Haberman (1961), and with the value of -0.79682417 obtained by Brenner & Sonshine (1964) for a point couple about the Z-axis.

Table 1. Tabulation of $A(\beta)$, $B(\beta)$ and $C(\beta)$ vs β

β	$A(\beta)$	$B(\beta)$	$C(\beta)$
0·00	−1.47111	−1.47111	−0.796811
0.01	−1.47159	−1.47197	−0.797304
0·03	−1.47549	−1.47887	−0.801257
0.05	−1.48334	−1.49274	−0.809207
0.10	−1.52078	−1.55886	−0.847258
0.15	−1.58575	−1.67327	−0.913679
0.20	−1.68248	−1.84289	−1.01342
0.25	−1.81768	−2.07875	−1.15440
0.30	−2.00156	−2.39773	−1.34883
0.35	−2.24960	−2.82548	−1.61539
0.40	−2.58550	−3.40159	−1.98309
0.45	−3.04643	−4.18858	−2.49808
0.50	−3.69281	−5.28882	−3.23651
0.55	−4.62721	−6.87763	−4.32971
0.60	−6.03374	−9.27264	−6.01729
0.65	−8.26681	−13.0913	−8.76846
0.70	−12.0725	−19.6473	−13.5888
0.75	−19.2185	−32.0870	−22.9069
0.80	−34.637	−59.293	−43.643
0.85	−75.886	−133.27	−101.01
0.86	−91.897	−162.24	−123.68
0.88	−171	−279	−196
0.90	−256	−422	−330

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